

Scalar fields, bent branes, and RG flow

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ABSTRACT: This work deals with braneworld scenarios driven by real scalar fields with standard dynamics. We show how the first-order formalism which exists in the case of four dimensional Minkowski space-time can be extended to de Sitter or anti-de Sitter geometry in the presence of several real scalar fields. We illustrate the results with some examples, and we take advantage of our findings to investigate renormalization group flow. We have found symmetric brane solutions with four-dimensional anti-de Sitter geometry whose holographically dual field theory exhibits a weakly coupled regime at high energy.

KEYWORDS: D-branes, Renormalization Group, Large Extra Dimensions.

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1. Introduction

In the present work, we focus attention on braneworld models described in five-dimensional space-time with warped geometry involving a single infinitely large extra dimension, as firstly introduced in ref. [1]. Following the original work of Randall and Sundrum, the braneworld scenario that we consider appears with five-dimensional AdS (or even Minkowski) geometry, and the 3-brane is now allowed to have four-dimensional space-time with AdS, Minkowski, or dS geometry [2–16].

The model describes five-dimensional gravity in the presence of dynamical bulk scalar fields. Our motivation arises from the investigations [3, 4, 11–13, 17–19], which consider the possibility of finding first-order differential equations which solve the corresponding equations of motion. This is technically important, since it directly contributes to simplify investigations, and to open new scenarios. The present study is connected with the former work [18] and inspired in similar calculations done by some of us in cosmology [17]. It is interesting to see that, although supersymmetry seem to be incompatible with dS geometry, even in this case we have found a way to write first-order equations which solve the equations of motion of the original system [18]. See also ref. [20] for another procedure, based on the Hamilton-Jacobi formalism.

The investigations focus on Einstein's equation and the equations of motion for the scalar fields in a very direct way. We consider models described by real scalar fields in five-dimensional space-time with anti-de Sitter (AdS), or Minkowski (\mathbb{M}) geometry, which

engenders a single extra dimension and generic four dimensional space-time with AdS, Minkowski, or dS geometry. The scalar fields are described with standard dynamics, and we follow a very specific route, set forward in [17], in which we use the potential of the scalar field to infer how the warp factor depends on the extra dimension.

The power of the method that we develop is related to an important simplification, which leads to models governed by scalar field potential of very specific form, depending on two new functions, $W = W(\phi)$ and $Z = Z(\phi)$. As we show below, we relate the function $W(\phi)$ to the warp factor, and this leads to scenarios where the scalar field may be connected with W and Z , unveiling a new route to investigate the subject.

We illustrate our findings with several examples of current interest, described by two real scalar fields, and we take advantage of our findings to investigate renormalization group flow for flat branes [21, 22, 3] and “bent” (or “curved”) branes [23]. We have found symmetric bent brane solutions with four-dimensional AdS geometry whose holographically dual field theory exhibits a weakly coupled regime at high energy. Such bent branes may give a dual gravitational description of RG flows in supersymmetric field theories living in the curved spacetime of the brane world-volume [23]. Furthermore, such AdS_4 brane theories exhibit improved infrared behavior. Other examples include *asymmetric branes* that are asymptotically $M_5 - AdS_5$ spaces. We find that in these theories there exists a ‘natural’ UV cut-off on the running coupling in the Minkowski (M_5) side.

The paper is organized as follows. In section 2 we look for flat and curved thick 3-brane solutions in a five-dimensional theory of gravity coupled with one scalar field. In section 3 we extend the earlier analysis to many scalar fields. In section 4 we investigate the localization of gravity for the solutions we find, and point out some issues in obtaining the calculations analytically. In section 5 we study the implications of the renormalization group flow for the dual field theory on the boundary of the five-dimensional spacetime. We make our final considerations in section 6.

2. One scalar field

The models that we investigate is described by a theory of five-dimensional gravity coupled to scalar fields governed by the following action

$$S = \int d^4x dy \sqrt{|g|} \left(-\frac{1}{4} R + \mathcal{L}(\phi, \partial_i \phi) \right), \tag{2.1}$$

where ϕ stands for a real scalar field and we are using $4\pi G = 1$. These theories with one or many scalar fields can simulate true five-dimensional supergravity theories under certain consistent truncations — see refs. [13, 19] for further discussions. The line element ds_5^2 of the five-dimensional space-time can be written as

$$ds_5^2 = g_{ij} dx^i dx^j = e^{2A} ds_4^2 - dy^2, \tag{2.2}$$

for $i, j = 0, 1, \dots, 4$. Also, ds_4^2 represents the line element of the four-dimensional space-time, which can have the form

$$ds_4^2 = dt^2 - e^{2\sqrt{\Lambda}t} (dx_1^2 + dx_2^2 + dx_3^2), \tag{2.3a}$$

$$ds_4^2 = e^{-2\sqrt{\Lambda}x_3} (dt^2 - dx_1^2 - dx_2^2) - dx_3^2, \tag{2.3b}$$

for dS or AdS geometry, respectively. Here e^{2A} is the warp factor and Λ represents the cosmological constant of the four-dimensional space-time; the limit $\Lambda \rightarrow 0$ leads to the line element

$$ds_5^2 = e^{2A} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2, \quad (2.4)$$

where $\eta_{\mu\nu}$, $\mu, \nu = 0, 1, 2, 3$, describes Minkowski geometry. The scalar field dynamics is governed by the Lagrangian density

$$\mathcal{L} = \frac{1}{2} g_{ij} \partial^i \phi \partial^j \phi - V, \quad (2.5)$$

where $V = V(\phi)$ represents the potential, which specifies the model to be considered.

2.1 Flat branes

Let us first focus on flat (or Minkowski) branes, i.e., the cases with $\Lambda = 0$. As usual, we suppose that both A and ϕ are static, and depend only on the extra dimension, that is, we set $A = A(y)$ and $\phi = \phi(y)$. In this case the equation of motion for the scalar field has the form

$$\phi'' + 4A'\phi' = V_\phi, \quad (2.6)$$

where prime denotes derivative with respect to y , and $V_\phi = dV/d\phi$. The Einstein's equation with Minkowski four-dimensional geometry gives

$$A'' = -\frac{2}{3}\phi'^2, \quad (2.7a)$$

$$A'^2 = \frac{1}{6}\phi'^2 - \frac{1}{3}V(\phi). \quad (2.7b)$$

To get to the first-order formalism [3–6], we introduce another function, $W = W(\phi)$, which can be viewed as a superpotential in supergravity extensions. By writing the first-order equation

$$A' = -\frac{1}{3}W, \quad (2.8)$$

and using the equation (2.7a) we get to

$$\phi' = \frac{1}{2}W_\phi, \quad (2.9)$$

and now the potential in eq. (2.7b) has the form

$$V = \frac{1}{8}W_\phi^2 - \frac{1}{3}W^2. \quad (2.10)$$

It is not difficult to show that eqs. (2.8) and (2.9) solve eqs. (2.6) and (2.7) for the above potential (2.10). We can also change $W \rightarrow -W$ to get another possibility without changing the potential. This result is very interesting, since it simplifies the calculation significantly. As one knows, it was already obtained in former works [3, 4].

In the following we illustrate the procedure with several flat brane examples. The first example is given by the superpotential [24, 25]

$$W = 2a \arctan(\sinh(b\phi)), \quad (2.11)$$

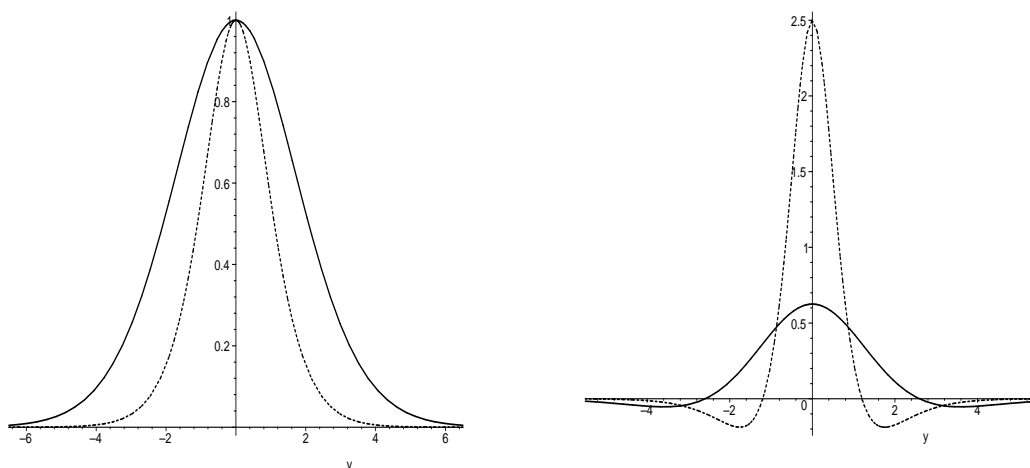


Figure 1: Warp factor (left panel) for $a = 1$, and $b = 1/2$ (solid line), $b = 1$ (dashed line), and the corresponding energy densities (right panel) for the scalar fields in curved space-time for the model described by eq. (2.11).

which gives the following scalar potential

$$V(\phi) = \frac{1}{2}a^2b^2\text{sech}^2(b\phi) - \frac{4}{3}a^2 \arctan^2(\sinh^2(b\phi)). \quad (2.12)$$

The solution of the first-order equations is

$$\phi(y) = \pm \frac{1}{b} \text{arcsinh}(ab^2y), \quad (2.13)$$

and

$$A(y) = \frac{1}{3b^2} \ln(1 + a^2b^4y^2) - \frac{2}{3}a y \arctan(ab^2y). \quad (2.14)$$

Another example is the well-known $\lambda\phi^4$ model obtained with

$$W = 2ab(\phi - b^2\phi^3/3), \quad (2.15)$$

which gives the scalar potential

$$V(\phi) = \frac{1}{2}a^2b^2(1 - b^2\phi^2)^2 - \frac{4}{3}a^2b^2\phi^2(1 - \frac{b^2}{3}\phi^2)^2. \quad (2.16)$$

For this model one finds

$$\phi(y) = \pm \frac{1}{b} \tanh(ab^2y), \quad (2.17)$$

and

$$A(y) = \frac{4}{9b^2} \ln(\text{sech}(ab^2y)) - \frac{1}{9b^2} \tanh^2(ab^2y). \quad (2.18)$$

In examples above a, b , are constants. Note that these two examples although they represent branes with finite energy, since their energy density are localized (figures 1 and 2, respectively) the kink profile behaves completely different at asymptotic limits. In the first example the kink connects vacua at infinity, whereas the well-known $\lambda\phi^4$ model connects the two vacua $\phi_{\text{vac}} = \pm 1/b$. In any case the vacua are supersymmetric since $W_\phi = 0$ on them.

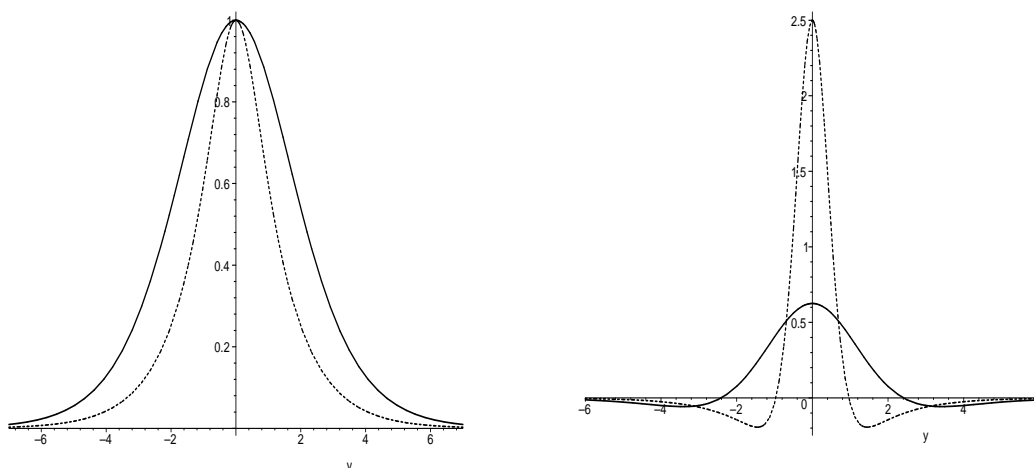


Figure 2: Warp factor (left panel) for $a = 1$, and $b = 1/2$ (solid line), $b = 1$ (dashed line), and the corresponding energy densities (right panel) for the scalar fields in curved space-time for the model described by eq. (2.15).

2.2 Bent branes

We now consider the general case of four dimensional AdS, or dS geometry. Here the Einstein's equation give

$$A'' + \Lambda e^{-2A} = -\frac{2}{3}\phi'^2, \quad (2.19a)$$

$$A'^2 - \Lambda e^{-2A} = \frac{1}{6}\phi'^2 - \frac{1}{3}V(\phi), \quad (2.19b)$$

for dS geometry ($\Lambda > 0$) or AdS geometry ($\Lambda < 0$). The case of Minkowski spacetime is obtained in the limit $\Lambda \rightarrow 0$, which leads us back to eqs. (2.7).

The presence of Λ makes the problem much harder. Interesting investigations have been already appeared in refs. [12, 13] and in references therein. Here, however, we follow another route. The key issue springs signaling for the need of a new constraint, and we suggest that

$$A' = -\left(\frac{1}{3}W + \frac{1}{3}\Lambda\gamma Z\right), \quad (2.20)$$

$$\phi' = \frac{1}{2}(W_\phi + \Lambda(\alpha + \gamma)Z_\phi), \quad (2.21)$$

where $Z = Z(\phi)$ is a new and in principle arbitrary function of the scalar field, to respond for the presence of the cosmological constant, and α, γ are constants. In this case the potential is given by

$$V = \frac{1}{8}(W_\phi + \Lambda(\alpha + \gamma)Z_\phi)(W_\phi + \Lambda(\gamma - 3\alpha)Z_\phi) - \frac{1}{3}(W + \Lambda\gamma Z)^2, \quad (2.22)$$

and we have to include the constraint

$$W_{\phi\phi}Z_\phi + W_\phi Z_{\phi\phi} + 2\Lambda(\alpha + \gamma)Z_\phi Z_{\phi\phi} - \frac{4}{3}Z_\phi(W + \Lambda\gamma Z) = 0, \quad (2.23)$$

to obtain

$$A(y) = -\frac{1}{2} \ln \left(\mp \frac{\alpha}{6} (W_\phi Z_\phi + \Lambda(\alpha + \gamma)Z_\phi^2) \right), \quad (2.24)$$

which opens diverse possibilities to obtain first-order formalism for braneworlds with nonzero cosmological constant. To illustrate this result, let us first consider the case $Z = \phi$, and

$$W = a \sinh(b\phi) - \Lambda\gamma\phi, \quad (2.25)$$

where, from eq. (2.23), $b = \pm 2/\sqrt{3}$, and we have the potential

$$V = \frac{1}{8} (ab \cosh(b\phi) + \Lambda\alpha) (ab \cosh(b\phi) - 3\Lambda\alpha) - \frac{1}{3} a^2 \sinh^2(b\phi). \quad (2.26)$$

In this case, the problem is solved with

$$\phi(y) = \pm \frac{2}{b} \operatorname{arctanh} \left(\sqrt{\frac{ab + \Lambda\alpha}{ab - \Lambda\alpha}} \tan \left(\frac{1}{4} b \sqrt{a^2 b^2 - \Lambda^2 \alpha^2} y \right) \right), \quad (2.27)$$

$$A(y) = -\frac{1}{2} \ln \left(\frac{\alpha}{6} \frac{a^2 b^2 - \Lambda^2 \alpha^2}{ab + \Lambda\alpha - 2ab \cos^2 \left(\frac{1}{4} b \sqrt{a^2 b^2 - \Lambda^2 \alpha^2} y \right)} \right). \quad (2.28)$$

The kink profile (2.27) and the brane geometry (2.28) have the main properties depicted in figures 3. For branes with AdS_4 geometry, i.e., $\Lambda < -ab/\alpha$ the kink is smooth, whereas for branes with dS_4 geometry, i.e., $\Lambda > ab/\alpha$, the kink becomes ‘singular’ in the sense that it diverges around $y^* = \pm 4 \operatorname{arctanh}[(ab - \Lambda\alpha)/\sqrt{-a^2 b^2 + \Lambda^2 \alpha^2}]/(b\sqrt{-a^2 b^2 + \Lambda^2 \alpha^2})$. See also, e.g., ref. [26], for another type of singularity in dS_4 branes. We assume $ab > 0$, $\alpha > 0$. The case $-ab/\alpha < \Lambda < ab/\alpha$, which necessarily includes branes with Minkowski geometry, i.e., $\Lambda = 0$, gives an array of singular kinks. This is a nice example where a singular brane with naked singularity (figure 3, *thin* line at right panel) can be smoothed out into another brane (figure 3, *thick* line at right panel) by turning on a negative cosmological constant on the brane.

Asymptotically the scalar field ϕ describing the *smooth* kink goes to the vacuum sector ϕ_{vac} (*finite* constant) and the scalar potential approaches the five-dimensional cosmological constant

$$V(\phi_{\text{vac}}) \equiv \Lambda_5 = -\frac{1}{3} (W(\phi_{\text{vac}}) + \Lambda\gamma Z(\phi_{\text{vac}}))^2 = -3A'(\pm\infty)^2 = \frac{a^2}{3} - \frac{\Lambda^2 \alpha^2}{4}, \quad (2.29)$$

where we have used $b = 2/\sqrt{3}$, the potential (2.26), and the solution (2.27)–(2.28) at asymptotic limits. Note that these are supersymmetric vacua since they satisfy $\phi'_{\text{vac}} = \pm(1/2)(W_\phi + \Lambda(\alpha + \gamma)Z_\phi)|_{\text{vac}} = 0$ (Recall that for flat branes, i.e., $\Lambda = 0$, the supersymmetric vacua satisfy the simple condition $W_\phi = 0$.) Furthermore they correspond to an asymptotic AdS_5 geometry, i.e., $\Lambda_5 < 0$, because $\alpha^2 \Lambda^2 > a^2 b^2$.

We end this section by considering another model obtained with $Z = W$ and superpotential

$$W = a \sinh(b\phi), \quad (2.30)$$

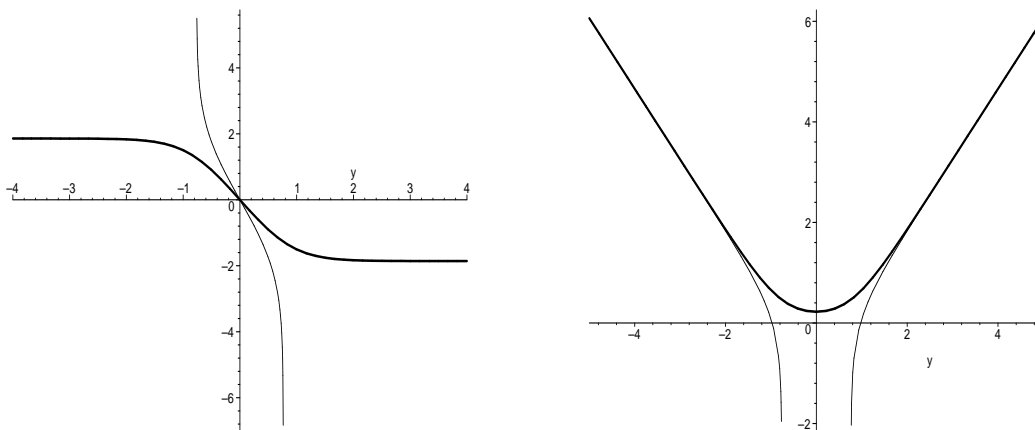


Figure 3: The kink profile (left panel) and $A(y)$ (right panel) are singular (thin line) for $\Lambda_{AdS} > ab/\alpha$ and non-singular for $\Lambda_{AdS} < -ab/\alpha$ (thick line) where $a = 1, b = 2/\sqrt{3}, \alpha = 1$.

where, from eq. (2.23), we have $b = \pm\sqrt{6(1 - \Lambda\alpha)(1 + \Lambda(\alpha + \gamma))/3(1 + \Lambda(\alpha + \gamma))}$. Here, we find the following scalar potential

$$V = \frac{1}{12}(1 - \Lambda\alpha)(1 + \Lambda(\gamma - 3\alpha)a^2 \cosh^2(b\phi) - \frac{1}{3}a^2(1 + \Lambda\gamma)^2 \sinh^2(b\phi). \quad (2.31)$$

In this case, the problem is solved with

$$\phi(y) = \pm\frac{1}{b}\operatorname{arcsinh}\left(\tan\left(\frac{1}{3}a(1 - \Lambda\alpha)y\right)\right), \quad (2.32)$$

$$A(y) = -\frac{1}{2}\ln\left(-\frac{1}{9}\alpha a^2(1 - \Lambda\alpha)\sec^2\left(\frac{1}{3}a(1 - \Lambda\alpha)y\right)\right). \quad (2.33)$$

Note that the periodic kink (2.32) is singular and the metric has naked singularity at $y^* = \pm 3\pi/2a(1 - \Lambda\alpha)$, for any cosmological constant Λ . For $\alpha > 0$, we only have dS_4 branes, since only positive cosmological constant are allowed in this case, and for $\alpha < 0$ we may have AdS_4 or dS_4 branes, because the cosmological constant on the brane can assume the values $-1/|\alpha| < \Lambda < 0$ or $\Lambda > 0$, respectively. Similar models have been first introduced in refs. [5, 6, 18].

3. Two scalar fields

We now extend the above procedure to the case of two or more real scalar fields with standard dynamics. We first investigate the important case where the brane has 4d Minkowski geometry and later we extend the analysis to 4d AdS and dS geometries. Here we have to change the Lagrangian density to the form

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \frac{1}{2}\partial_\mu\rho\partial^\mu\rho + \dots + \frac{1}{2}\partial_\mu\zeta\partial^\mu\zeta - V(\phi, \chi, \rho, \dots, \zeta). \quad (3.1)$$

3.1 Flat branes

For flat brane geometry in many fields theory we get the new set of equations

$$\phi'' + 4A'\phi' = V_\phi, \quad \chi'' + 4A'\chi' = V_\chi, \quad \rho'' + 4A'\rho' = V_\rho, \quad \dots, \quad \zeta'' + 4A'\zeta' = V_\zeta \quad (3.2a)$$

$$A'' = -\frac{2}{3}\phi'^2 - \frac{2}{3}\chi'^2 - \frac{2}{3}\rho'^2 - \dots - \frac{2}{3}\zeta'^2, \quad (3.2b)$$

$$A'^2 = \frac{1}{6}\phi'^2 + \frac{1}{6}\chi'^2 + \frac{1}{6}\rho'^2 + \dots + \frac{1}{6}\zeta'^2 - \frac{1}{3}V. \quad (3.2c)$$

As before, we insist with $A' = -W/3$, but now $W = W(\phi, \chi, \rho, \dots, \zeta)$ suggests that we write the two first-order equations

$$\phi' = \frac{1}{2}W_\phi, \quad \chi' = \frac{1}{2}W_\chi, \quad \rho' = \frac{1}{2}W_\rho, \quad \dots, \quad \zeta' = \frac{1}{2}W_\zeta, \quad (3.3)$$

and the potential is now given by

$$V = \frac{1}{8}W_\phi^2 + \frac{1}{8}W_\chi^2 + \frac{1}{8}W_\rho^2 + \dots + \frac{1}{8}W_\zeta^2 - \frac{1}{3}W^2. \quad (3.4)$$

It is not hard to show that solutions of the above first-order equations also solve the set of eqs. (3.2) for the potential (3.4). The above procedure opens interesting possibilities for setups of coupled fields and branes with flat geometry.

In the case of two scalar fields, for instance, if we consider an additive W , that is, if we take $W(\phi, \chi) = W_1(\phi) + W_2(\chi)$, we get the potential in the form $V(\phi, \chi) = V_1(\phi) + V_2(\chi) - (2/3)W_1(\phi)W_2(\chi)$. It shows that the interactions appear as the product of the two independent W_1 and W_2 . As an example we consider

$$W = 3a \sin(b\phi) + 3c \sinh(d\chi), \quad (3.5)$$

where a, b, c, d are constants. The scalar potential for such superpotential is

$$V = \frac{9}{8}a^2b^2 \cos^2(b\phi) + \frac{9}{8}c^2d^2 \cosh^2(d\chi) - 3(a \sin(b\phi) + c \sinh(d\chi))^2, \quad (3.6)$$

such that the solution of the decoupled first order equations reads

$$\phi(y) = \pm \frac{1}{b} \arcsin[\tanh(\frac{3}{2}ab^2y)], \quad (3.7a)$$

$$\chi(y) = \pm \frac{1}{d} \operatorname{arcsinh}[\tan(\frac{3}{2}cd^2y)], \quad (3.7b)$$

and

$$A(y) = -\frac{2}{3b^2} \ln[q \cosh(\frac{3}{2}ab^2y)] - \frac{2}{3d^2} \ln[p \sec(\frac{3}{2}cd^2y)], \quad (3.8)$$

where p and q are real positive constants. In this case, the metric has naked singularity at $y^* = \pm\pi/3cd^2$. The case where $d = 0$ or $b = 0$ reduces to the case of one scalar field system, as it has been previously found in refs. [5, 6] and [18], respectively. The warp factor e^{2A} and energy density for this solution are depicted in figure 4. Note that the case $c = 0$ is equivalent to have only the field ϕ , and there is no naked singularity on the metric. By considering only the scalar field ϕ component, i.e., $c = 0$, the vacuum is achieved in

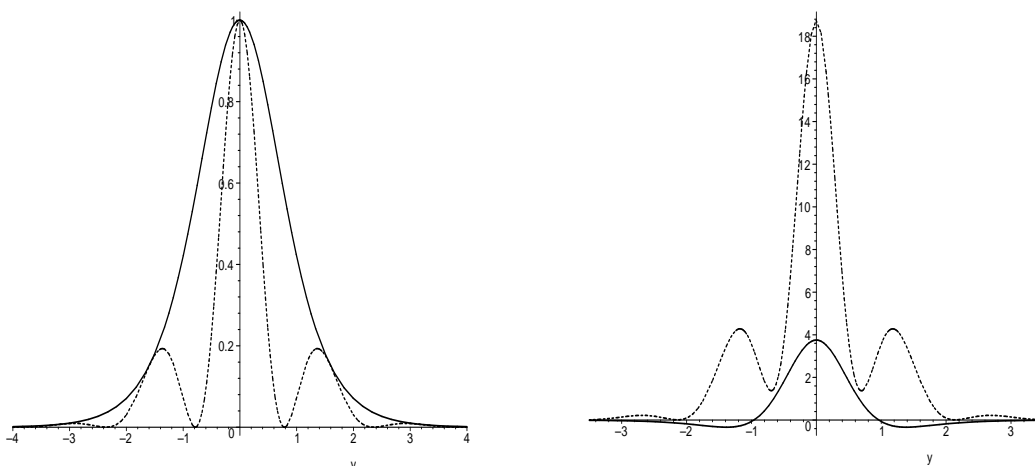


Figure 4: Warp factor (left panel) for $c = 0$, and 2 (solid line, and dashed line) and the corresponding energy densities (right panel) for the scalar fields in curved space-time for the model described by eq. (3.5) with $b = d = \sqrt{2/3}$ and $a = p = q = 1$.

the asymptotic limits $\phi_{\text{vac}} = \phi(\pm\infty)$. The metric asymptotically describes an AdS_5 space whose cosmological constant is $\Lambda_5 \equiv V(\phi_{\text{vac}}) = -3a^2$. On the other hand, by turning on both scalar field components, i.e., $a, b, c, d \neq 0$, the solution becomes singular and periodic.

Another possibility, that couples the scalar fields in the superpotential, is given by

$$W = 3a \sin(b\phi) \cos(b\chi). \tag{3.9}$$

In this case we have

$$V = \frac{9}{8}a^2b^2 (\cos^2(b\phi) \cos^2(b\chi) + \sin^2(b\phi) \sin^2(b\chi)) - 3a^2 \sin^2(b\phi) \cos^2(b\chi). \tag{3.10}$$

We can find the solution of the coupled first order equations by using the orbits $\cos(b\phi) = C \sin(b\chi)$ being C a real constant. For $C = 0$ we have

$$\phi(y) = (2m + 1)\frac{\pi}{2b}, \quad \chi(y) = \pm \frac{1}{b} \arccos \left(\tanh \left(\frac{3}{2}ab^2y \right) \right) + k \frac{\pi}{b}, \tag{3.11}$$

or

$$\phi(y) = \pm \frac{1}{b} \arcsin \left(\tanh \left(\frac{3}{2}ab^2y \right) \right) + k \frac{\pi}{b}, \quad \chi(y) = m \frac{\pi}{b}, \tag{3.12}$$

where m and k are integer, from that

$$A(y) = -\frac{2}{3b^2} \ln [q \cosh(\frac{3}{2}ab^2y)], \tag{3.13}$$

where the constant $q > 0$. The warp factor e^{2A} and energy density for this solution is shown in figure 5. These brane solutions are supersymmetric in the sense that asymptotically, i.e., at the vacuum $\phi_{\text{vac}} = \phi(y = \pm\infty), \chi_{\text{vac}} = \chi(y = \pm\infty)$, we find $W_\phi, W_\chi = 0$. The

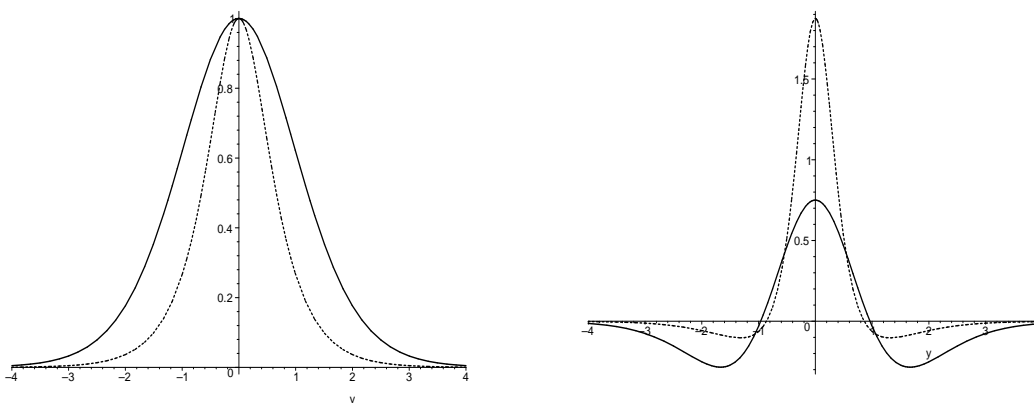


Figure 5: Warp factor (left panel) with $a = 1$ and $b = 1/\sqrt{3}$ (solid line), $b = 2/\sqrt{3}$ (dashed line), and the corresponding energy density (right panel)

bulk is asymptotically a five-dimensional AdS_5 space-time whose cosmological constant is $\Lambda_5 \equiv V(\phi_{vac}, \chi_{vac}) = -3a^2$.

For $C = 1$ we have

$$\phi(y) = \pm \frac{1}{2b} \arccos \left(\tanh \left(\frac{3}{4} ab^2 y \right) \right) + (k+1) \frac{\pi}{2b}, \quad \chi(y) = \pm \frac{1}{2b} \arccos \left(\tanh \left(\frac{3}{2} ab^2 y \right) \right) + k \frac{\pi}{2b}, \quad (3.14)$$

from that

$$A(y) = (-1)^{k+1} \frac{ay}{2} + \frac{2}{3b^2} \ln \left[q \operatorname{sech} \left(\frac{3}{4} ab^2 y \right) \right], \quad (3.15)$$

where the constant $q > 0$. Now we have asymmetric (figure 6) and symmetric (figure 7) branes. Let us now concern about the rich vacuum structure and geometry of these solutions. The supersymmetric vacua satisfying $W_\phi = 0$, $W_\chi = 0$ are connected by BPS and non-BPS branes. The superpotential $W(\phi, \chi)$ evaluated at the vacua (ϕ_{vac}, χ_{vac}) gives the asymptotic behavior of the geometry governed by equation $A' = -W/3$. At the vacua, i.e., for $y = \pm\infty$, the kink solutions for k odd or even give us (i) $W_{odd}^+ = 0$, (ii) $W_{odd}^- = 3a$, (iii) $W_{even}^+ = 3a$ and (iv) $W_{even}^- = 0$. At these supersymmetric vacua the 5d cosmological constant $\Lambda_5 \equiv V(\phi_{vac}, \chi_{vac}) = -(1/3)[W_{even/odd}^\pm]^2 \leq 0$ asymptotically characterizes five-dimensional Minkowski (\mathbb{M}_5) or anti-de Sitter (AdS_5) spaces. The cases (i)-(ii) and (iii)-(iv) describe *asymmetric* branes connecting asymptotically $AdS_5 - \mathbb{M}_5$ spaces and $\mathbb{M}_5 - AdS_5$ spaces, respectively — See figure 6. For further discussions on asymmetric branes see, e.g., refs. [27–31]. On the other hand, we can patch together *even* and *odd* solutions to form \mathbb{Z}_2 symmetric branes. The cases (ii)-(iii) and (i)-(iv) describe such *symmetric* branes connecting asymptotically $AdS_5 - AdS_5$ spaces and $\mathbb{M}_5 - \mathbb{M}_5$ spaces, respectively — See figure 7. These symmetric branes are clearly non-BPS branes, because the BPS bound [32–36] $\sigma_{BPS} = |\Delta W| = |W_{even/odd}^\pm - W_{odd/even}^\mp| = 0$, in contrast with the asymmetric BPS branes whose BPS bound $\sigma_{BPS} = 3a$. Locally, the geometry of the branes around $y = 0$ behaves according to the following branches $A(y) \simeq (-1)^{k+1} ay/2$. We can

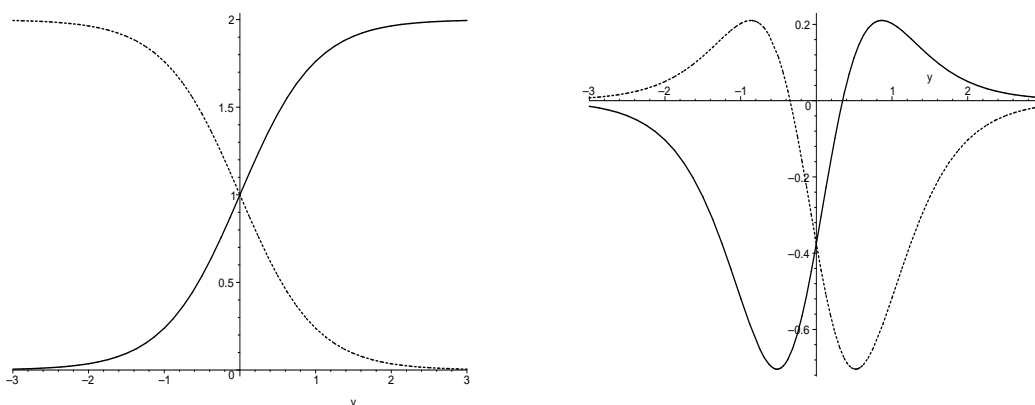


Figure 6: Warp factor of asymmetric BPS branes connecting asymptotically $AdS_5 - M_5$ spaces (solid line) and $M_5 - AdS_5$ spaces (dashed line), with $a = 1$ and $b = 2/\sqrt{3}$ (left panel), and the corresponding energy densities (right panel).

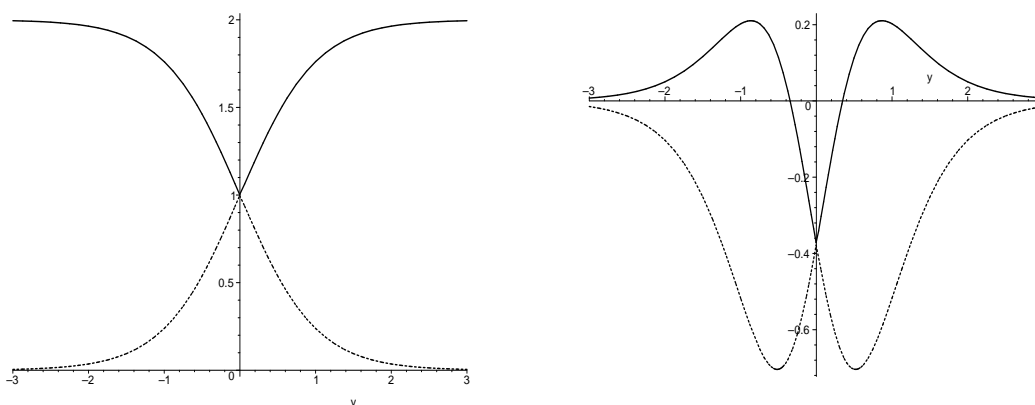


Figure 7: Warp factor of symmetric non-BPS branes connecting asymptotically $M_5 - M_5$ spaces (solid line) and $AdS_5 - AdS_5$ spaces (dashed line), with $a = 1$ and $b = 2/\sqrt{3}$ (left panel), and the corresponding energy densities (right panel).

patch together two local branches along with the symmetric branes $A(y) \simeq -a|y|/2$ (see warp factor in figure 7 — dashed line) and $A(y) \simeq a|y|/2$ (see warp factor in figure 7 — solid line).

3.2 Bent branes

We now consider the general case of branes with four dimensional anti-de Sitter (AdS_4) or de Sitter (dS_4) geometry. Let us restrict ourselves to a two scalar field theory on the

background (2.2). The Einstein's equations give

$$A'' + \Lambda e^{-2A} = -\frac{2}{3}\phi'^2 - \frac{2}{3}\chi'^2, \quad (3.16a)$$

$$A'^2 - \Lambda e^{-2A} = \frac{1}{6}\phi'^2 + \frac{1}{6}\chi'^2 - \frac{1}{3}V(\phi, \chi), \quad (3.16b)$$

for dS_4 ($\Lambda > 0$) or AdS_4 ($\Lambda < 0$) geometry. The case of Minkowski space-time is obtained in the limit $\Lambda \rightarrow 0$, which leads us back to eqs. (3.2).

The presence of the four dimensional cosmological constant Λ makes the problem much harder, but it can be done following the same way employed for one scalar field. In this sense, we suggest that the problem of integrating second-order equation of motion can be reduced to the set of first-order equations

$$A' = -\frac{1}{3}(W + \Lambda\gamma Z), \quad (3.17)$$

$$\phi' = \frac{1}{2}(W_\phi + \Lambda(\alpha + \gamma) Z_\phi), \quad (3.18)$$

$$\chi' = \frac{1}{2}(W_\chi + \Lambda(\beta + \gamma) Z_\chi), \quad (3.19)$$

where $Z = Z(\phi, \chi)$ is a new and in principle arbitrary function of the scalar field, to respond for the presence of the cosmological constant, and α, β, γ are constants. The suggestion (3.17)–(3.19) is consistent with eqs. (3.16a)–(3.16b) if the scalar potential is given by

$$V(\phi, \chi) = \frac{1}{8}(W_\phi + \Lambda(\alpha + \gamma)Z_\phi)(W_\phi + \Lambda(\gamma - 3\alpha)Z_\phi) + \frac{1}{8}(W_\chi + \Lambda(\beta + \gamma)Z_\chi)(W_\chi + \Lambda(\gamma - 3\beta)Z_\chi) - \frac{1}{3}(W + \Lambda\gamma Z)^2, \quad (3.20)$$

and if we impose the following constraints

$$\alpha W_\phi Z_{\phi\phi} + \alpha Z_\phi W_{\phi\phi} + 2\Lambda\alpha(\alpha + \gamma)Z_\phi Z_{\phi\phi} + \frac{1}{2}(\alpha + \beta)W_\chi Z_{\phi\chi} + \beta Z_\chi W_{\phi\chi} + \frac{1}{2}\Lambda(\beta + \gamma)(\alpha + 3\beta)Z_\chi Z_{\phi\chi} - \frac{4}{3}\alpha Z_\phi(W + \Lambda\gamma Z) = 0, \quad (3.21a)$$

$$\beta W_\chi Z_{\chi\chi} + \beta Z_\chi W_{\chi\chi} + 2\Lambda\beta(\beta + \gamma)Z_\chi Z_{\chi\chi} + \frac{1}{2}(\alpha + \beta)W_\phi Z_{\phi\chi} + \alpha Z_\phi W_{\phi\chi} + \frac{1}{2}\Lambda(\alpha + \gamma)(3\alpha + \beta)Z_\phi Z_{\phi\chi} - \frac{4}{3}\beta Z_\chi(W + \Lambda\gamma Z) = 0. \quad (3.21b)$$

After such considerations we obtain

$$A(y) = -\frac{1}{2} \ln \left(-\frac{\alpha}{6} (W_\phi Z_\phi + \Lambda(\alpha + \gamma)Z_\phi^2) - \frac{\beta}{6} (W_\chi Z_\chi + \Lambda(\beta + \gamma)Z_\chi^2) \right). \quad (3.22)$$

To illustrate this new result, let us consider the case $Z = W$, with $\gamma = 0$ and $\beta = \alpha$. We take

$$W = 3a \sin(b\phi + c\chi), \quad (3.23)$$

where a , b , and c are real constants with $b^2 + c^2 = -2/3(1 + \Lambda\alpha)$, and the scalar potential reads

$$V(\phi, \chi) = 3a^2 \left(\left(1 - \frac{1}{4}(1 - 3\Lambda\alpha) \right) \cos^2(b\phi + c\chi) - 1 \right). \quad (3.24)$$

This potential has global and local minima. As $(1 - 3\Lambda\alpha) < 4$ there exist global minima given by $b\phi + c\chi = \pm(2m + 1)\pi/2$ and for $(1 - 3\Lambda\alpha) > 4$, there exist local minima at $b\phi + c\chi = \pm m\pi$, where $m = 0, 1, 2, 3, \dots$. It is not difficult to notice that the global minima are supersymmetric vacua because $W_\phi = Z_\phi = 0$ and $W_\chi = Z_\chi = 0$ implies $b\phi_{\text{vac}} + c\chi_{\text{vac}} = \pm(2m + 1)\pi/2$. The first-order equations have solutions for the orbits $b\chi = c\phi + C$, where C is a real constant. For $(1 - 3\Lambda\alpha) < 4$ and $C = b/c n\pi$, or $(1 - 3\Lambda\alpha) > 4$ and $C = b/c (2n + 1)\pi/2$, we have the regular kinks

$$\phi(y) = \pm \frac{1}{d} \arcsin(\tanh(ay)) + k \frac{\pi}{d}, \quad (3.25)$$

and the irregular kinks

$$\phi(y) = \pm \frac{1}{d} \arcsin(\coth(ay)) + k \frac{\pi}{d}. \quad (3.26)$$

Furthermore, for $(1 - 3\Lambda\alpha) < 4$ and $C = b/c (2n + 1)\pi/2$, or $(1 - 3\Lambda\alpha) > 4$ and $C = b/c n\pi$, we have the regular kinks

$$\phi(y) = \pm \frac{1}{d} \arccos(\tanh(ay)) + k \frac{\pi}{d}, \quad (3.27)$$

and the irregular kinks

$$\phi(y) = \pm \frac{1}{d} \arccos(\coth(ay)) + k \frac{\pi}{d}, \quad (3.28)$$

where k and n are integer numbers and $d = -2/b(1 + \Lambda\alpha)$. For regular kink solutions we obtain

$$A(y) = \ln \left[\sqrt{\frac{1}{\alpha}} \frac{1}{|a|} \cosh(ay) \right], \quad (3.29)$$

with $\alpha > 0$. Since $\alpha = (-\Lambda)^{-1}[2/3(b^2 + c^2) + 1]$, $\Lambda < 0$, that is, this solution represents a brane with AdS_4 geometry. On the other hand, for irregular kink solutions we obtain

$$A(y) = \ln \left[\sqrt{\frac{1}{-\alpha}} \frac{1}{|a|} |\sinh(ay)| \right], \quad (3.30)$$

with $\alpha < 0$. Now, because $\alpha = (-\Lambda)^{-1}[2/3(b^2 + c^2) + 1]$, $\Lambda > 0$, this solution represents a brane with dS_4 geometry.

It is instructive to notice that for two strongly coupled fields, i.e., $b^2 + c^2 \gg 1$ we find that $\alpha = (-\Lambda)^{-1}$. Furthermore, at the global (or supersymmetric) vacua we find $W(\phi_{\text{vac}}, \chi_{\text{vac}}) = 3a$ and then the 5d cosmological constant is $\Lambda_5 \equiv V(\phi_{\text{vac}}, \chi_{\text{vac}}) = -(1/3)W^2 = -3a^2 = -3/L^2$, where we have identified $a = 1/L$, being L the AdS_5 radius.

Under such considerations the solution (3.29) reduces to the familiar solution of a brane with AdS_4 geometry [4, 10, 33] (here, however, there is no δ -source for the brane):

$$A(y) = \ln \left[\sqrt{-\Lambda} L \cosh \left(\frac{ay}{L} \right) \right]. \quad (3.31)$$

Similarly, the solution of a brane with dS_4 (3.30) geometry can be written in the familiar form

$$A(y) = \ln \left[\sqrt{\Lambda} L \sinh \left(\frac{ay}{L} \right) \right]. \quad (3.32)$$

Although on one hand it is hard to find explicit solutions for many scalar fields, on the other hand it is straightforward to generalize the formalism above for N scalar fields. The scalar potential is given by

$$V(\phi_1, \dots, \phi_N) = \frac{1}{8} \sum_{i=1}^N \left(\partial_i W + \Lambda(\alpha_i + \gamma) \partial_i Z \right) \left(\partial_i W + \Lambda(\gamma - 3\alpha_i) \partial_i Z \right) - \frac{1}{3} (W + \Lambda\gamma Z)^2, \quad (3.33)$$

and the first-order equations read

$$\begin{aligned} \phi'_i &= \pm \frac{1}{2} \partial_i \left(W + \Lambda(\alpha_i + \gamma) Z \right), \quad i = 1, 2, \dots, N \\ A' &= \mp \frac{1}{3} (W + \Lambda\gamma Z). \end{aligned} \quad (3.34)$$

The constraint equations can be now written as

$$\partial_i (\partial_i W \partial_i Z) + \dots + \left[2\Lambda(\alpha_i + \gamma) \partial_i \partial_i Z - \frac{4}{3} (W + \Lambda\gamma Z) \right] \partial_i Z = 0, \quad i = 1, 2, \dots, N. \quad (3.35)$$

4. Gravity localization

The study of gravity localization on the brane solutions above can be done by choosing a gauge where the general metric fluctuations have the form

$$ds^2 = e^{2A(y)} (g_{\mu\nu} + \epsilon h_{\mu\nu}) dx^\mu dx^\nu - dy^2. \quad (4.1)$$

Here $g_{\mu\nu} = g_{\mu\nu}(x, y)$ represents the four-dimensional dS, AdS or Minkowski metric, and $h_{\mu\nu} = h_{\mu\nu}(x, y)$ represents the metric fluctuations, and ϵ is a small parameter. Following the refs. [4, 10, 7, 8], introducing the z -coordinate, in order to turn the metric conformally flat, with $dz = e^{-A(y)} dy$, the metric fluctuations of the brane solutions, under the choice of transverse and traceless gauge, leads to the Schroedinger-like equation

$$-\frac{d^2\psi(z)}{dz^2} + U(z)\psi(z) = m^2\psi(z), \quad (4.2)$$

with

$$U(z) = \frac{9}{4} A'^2(z) + \frac{3}{2} A''(z), \quad (4.3)$$

for dS, AdS or Minkowski geometry. Note that this equation can be factorized as

$$\left[-\frac{d}{dz} + \frac{3}{4}A'(z)\right] \left[\frac{d}{dz} + \frac{3}{4}A'(z)\right] \psi(z) = m^2\psi(z). \quad (4.4)$$

This shows that there are no graviton bound-states with negative mass, and the graviton zero mode $\psi_0(z) = e^{-\frac{3}{4}A(z)}$ is the ground-state of the quantum mechanical problem.

Before going into an explicit example several comments are in order. Due to difficulty in obtaining $A(z)$ from $A(y)$ in some brane solutions we have previously considered the calculation of the graviton spectrum on the brane may require numerical computations. For example, the study of the metric fluctuations via eq. (4.2) of the brane solutions defined by the superpotentials (2.11), eq. (2.15), and eq. (2.25), is only tractable numerically. The case defined by eq. (2.18) has been already done in the paper [37], for $a = b = 1$. The model defined by the superpotential (2.30) leads to a Schroedinger-like equation (4.2) whose potential is the modified Poschl-Teller type potential, and was studied in ref. [6, 18]. The two-field model given by the superpotential (3.5) is only tractable numerically. However, for $b = 0$ it reduces to the one-field model $W = 3c \sinh(d\chi)$, that for $d = \pm\sqrt{1/3}$ leads to a volcano-like potential, whose spectrum has been discussed in refs. [4–8], and for $d = \pm\sqrt{2/3}$, the spectrum was investigated in refs. [6, 18]. The case $d = 0$ gives the model $W = 3a \sin(b\phi)$ whose spectrum is tractable analytically for $b = \pm\sqrt{1/3}$ and $b = \pm\sqrt{2/3}$ — see ref. [5, 6]. The model defined by the superpotential (3.9) gives $A(y)$ of eq. (3.13) and eq. (3.15). The case (3.13), leads to the same situation of eq. (3.8) for $d = 0$, whereas for the case eq. (3.15), the gravity fluctuations are only tractable numerically.

Finally, we consider explicitly the two-field model for $\Lambda \neq 0$ with the superpotential (3.23) that can be studied analytically. From the AdS_4 solution eq. (3.29) we have

$$A(z) = -\frac{1}{2} \ln \left(\alpha a^2 \cos^2 \left(\frac{z}{\sqrt{\alpha}} \right) \right). \quad (4.5)$$

The Schroedinger-like potential (4.3) is now given by

$$U(z) = -\frac{9}{4\alpha} + \frac{15}{4\alpha} \sec^2 \left(\frac{z}{\sqrt{\alpha}} \right). \quad (4.6)$$

This is a Poschl-Teller potential. The model supports an infinity of bound states with eigenvalues given by

$$m_n^2 = \frac{n}{\alpha} (n + 3), \quad n = 1, 2, 3, \dots \quad (4.7)$$

where $\alpha = (-\Lambda)^{-1}[2/3(b^2 + c^2) + 1]$, $\Lambda < 0$. This potential is the same as the potential found in Karch-Randall scenario [10]. The spectrum consists of massive graviton modes of the gravity fluctuations of a pure AdS_5 spacetime. The gravity localization on the 3-brane is due to a very light mode that appears as the brane tension becomes sufficiently large [10, 38–41]. Although the brane solution of the model (3.23) has a nonzero tension, e.g., $\sigma_{\text{BPS}} = |\Delta W| = 3a$, for $m = k = 0$, such information does not appear in the potential (4.6). This makes impossible to control the spectrum (4.7) in order for the lightest graviton mode responsible for 4d gravity to emerge. If on one hand it is hard to

localize gravity with this brane solution, on the other hand, as we investigate the RG group flow later, it shows to be a nice *gravity dual* of a weakly coupled field theory on the AdS_5 boundary.

5. RG flow equations

According to gauge/gravity duality conjecture such as AdS/CFT correspondence [42–44] or domain wall/QFT correspondence [21] there exists the possibility of considering the warp factor of a spacetime geometry as a scale of energy of a holographically dual field theory on its boundary. In this section we are going to consider such conjecture by exploring the renormalization group flow of the dual field theory. Let us write the geometry (2.2) as

$$ds_5^2 = U^2(y)ds_4^2 - dy^2, \tag{5.1}$$

where $U(y) = e^{A(y)}$. As such, the warp factor is identified with the renormalization scale U on the flow equations [3, 21–23].

We first consider the case of a single scalar field. We write

$$\phi' = \frac{d\phi}{dy} = \frac{dU}{dy} \frac{d\phi}{dU} = A'U \frac{d\phi}{dU}. \tag{5.2}$$

If the scalar fields on the gravity side is conjectured to be related to running couplings on the dual field theory side we can use eq. (5.2) to construct the following beta function

$$\beta(\phi) \equiv U \frac{d\phi}{dU} = \frac{\phi'}{A'} = -\frac{3}{2} \frac{W_\phi + \Lambda(\alpha + \gamma)Z_\phi}{W + \Lambda\gamma Z}, \tag{5.3}$$

where we have used eqs. (2.20) and (2.21). Note that the beta function (5.3) works for both flat and bent branes supported by a single scalar field. At critical points $\phi = \phi^*$ (or $\phi = \phi_{\text{vac}}$ for supersymmetric vacua) the beta function vanishes. Thus, expanding the beta function $\beta(\phi)$ around the critical point we find

$$\beta(\phi) = \beta(\phi^*) + \beta'(\phi^*)(\phi - \phi^*) + \dots, \tag{5.4}$$

where $\beta(\phi^*) = 0$, and $\beta'(\phi^*)$ can be expressed in terms of W and Z as

$$\beta'(\phi^*) = -\frac{3}{2} \left[\frac{W_{\phi\phi} + \Lambda(\alpha + \gamma)Z_{\phi\phi}}{W + \Lambda\gamma Z} - \frac{(W_\phi + \Lambda(\alpha + \gamma)Z_\phi)(W_\phi + \Lambda\gamma Z_\phi)}{(W + \Lambda\gamma Z)^2} \right]_{\phi=\phi^*}. \tag{5.5}$$

Combining the equations (5.3) and (5.4) and integrating out both sides one can find the following running coupling equation

$$\phi = \phi^* + cU^{\beta'(\phi^*)}, \tag{5.6}$$

where c is a constant. For $\beta'(\phi^*) < 0$ and energy scale $U \rightarrow \infty$ we have that $\phi = \phi^*$ is an ultraviolet (UV) stable *fixed point*, whereas for $\beta'(\phi^*) > 0$ and energy scale $U \rightarrow 0$ we have that $\phi = \phi^*$ is an infrared (IR) stable *fixed point*. An AdS_5 vacuum solution $U = e^{ky}$ gives a (weak) strong running coupling ϕ as $y \rightarrow \infty$, i.e., $U \rightarrow \infty$, for ($\beta'(\phi^*) < 0$) $\beta'(\phi^*) > 0$.

Let us investigate our brane solutions whose kink profile can be identified with running coupling of the dual field theory. For the Z_2 -symmetric branes $U(y \rightarrow +\infty) = U(y \rightarrow -\infty)$ such that it is enough to focus just on one slice of the 5d spacetime, say, $U = e^{A(y)}$, ($y > 0$) at the vacuum ($y \rightarrow \infty$). It is interesting, from the AdS/CFT correspondence, those solutions which are asymptotically AdS_5 with $U(y \rightarrow \infty) = \infty$ (UV stable fixed point).

The $\lambda\phi^4$ example with $\Lambda = 0$ in (2.15) gives us $\beta'(\phi^*) = 9b^2/2$ which means there exists an IR stable fixed point on the dual field. This result signalizes gravity localization on the brane [45, 46].

On the other hand, the bent brane example with $\Lambda < 0$ in (2.25) gives us $\beta'(\phi^*) = -3b^2/2$ that implies the existence of an UV stable fixed point on the dual field. Thus, this field theory is a weakly coupled theory at high energy, although the coupling never diverges because the kink smoothly connects two different vacua with the same scale $U(y = \pm\infty) = \infty$. As a comparison, recall for the dilaton domain wall [21, 22], $U(y = -\infty) = 0$ and $U(y = \infty) = \infty$. In our bent brane solution we find that at $y = 0$, the smallest distance in the bulk at one side of the brane, the energy scale becomes $U = 1$. This signalizes that a non-confining phase in the infrared regime may appear. There is a “natural” IR cut-off in this space. Note that asymptotically, i.e., $y \rightarrow \pm\infty$, $A(y) \simeq \pm y/R$, one has AdS_5 slices. Now changing the coordinates of the metric (5.1) as $U = e^{A(y)} = r/R$, one finds

$$ds_5^2 = \frac{r^2}{R^2} ds_4^2 - \frac{R^2}{r^2} dr^2. \tag{5.7}$$

These AdS_5 slices are connected by the AdS_4 brane at $y = 0$. Of course, the range of the coordinate r for a slice, say, $y \geq 0$, is restricted to $R \leq r < \infty$, such that $r = R$ is an infrared cut-off of this AdS_5 slice. Thus the position of the AdS_4 brane at $y = 0$ (or equivalently at $r = R$) is a natural infrared cut-off. In recent developments [47, 48], in which one considers the introduction of IR cut-off to obtain a deconfining phase transition, one extends an AdS_5 metric like the metric (5.7) to an AdS-Schwarzschild metric in ten-dimensions, whose deconfining temperature is given in terms of a relation between the horizon radius and the infrared cut-off.

Note also that the brane in the case $\Lambda > 0$, because of its singular behavior, does not present a well defined beta function — same happens to the model (2.30). As we have earlier discussed, the negative cosmological constant also resolves the singularity of the brane at infrared regime — see figure 3.

Another interesting example is the one given in (2.11), whose kink profile connects vacua at infinity. It produces $\beta'(\phi^*) = 0$, which means from (5.6) that $\phi = \phi_*$ is fixed everywhere, that is, we have on the boundary, a dual conformal field theory.

The extension to a theory with multi-running couplings ϕ^i is straightforward. The equations (5.3) and (5.4) can be now combined in the form

$$\beta^i(\phi) \equiv U \frac{d\phi^i}{dU} = \frac{\partial\beta^i(\phi^*)}{\partial\phi^j} (\phi^j - \phi^{j*}) + \dots \tag{5.8}$$

where $\beta^i(\phi^*)$ is eq. (5.5) for multi-running couplings and $\partial\beta^i(\phi^*)/\partial\phi^j$ the corresponding derivatives. Now we are ready to discuss kink profiles of branes found in the two-field models that we have considered earlier.

The model $W = 3a \sin(b\phi) \cos(b\chi)$ with $\Lambda = 0$ – see eq. (3.9) – has the following derivative of the beta functions

$$\frac{\partial\beta^\phi(\phi^*)}{\partial\phi} = \frac{3}{2}b^2, \quad \frac{\partial\beta^\chi(\phi^*)}{\partial\chi} = \frac{3}{2}b^2, \quad \frac{\partial\beta^\phi(\phi^*)}{\partial\chi} = \frac{\partial\beta^\chi(\phi^*)}{\partial\phi} = -\frac{3}{2}b^2 \tan^2\left(\frac{k\pi}{2}\right) \quad (5.9)$$

This was done for $C = 1$. In the solution for $C = 0$, the second derivative adds to zero. Since k are integer numbers, the second derivative above is finite only if k is *even*. This is the case of the *asymmetric* supersymmetric $\mathbb{M}_5 - AdS_5$ brane discussed earlier. The running couplings equations are

$$\phi^i = \phi^{i*} + c^i U^{\frac{3}{2}b^2}, \quad \phi^i = (\phi, \chi). \quad (5.10)$$

In this model $U(y \rightarrow \infty) \rightarrow 0$ such that we have an IR stable fixed point $\phi^i = \phi^{i*}$. Note also that on the Minkowski side of the brane $U(y = -\infty) = \text{const} < \infty$ – see figure 6–, thus the running couplings have a ‘*natural*’ *UV cut-off*. The running couplings (ϕ, χ) vary their strengths in the same way.

Let us now finish this section by discussing the beta function of the ‘*bent*’ brane model $W = 3a \sin(b\phi + c\chi)$ with $\Lambda \neq 0$ and $Z = W$ – see eq. (3.23). For regular kink profile the cosmological constant $\Lambda < 0$ and supersymmetric vacua satisfy the relation $1 + \Lambda\alpha > 0$, $\alpha > 0$. The beta functions in this case obey

$$\frac{\partial\beta^\phi(\phi^*)}{\partial\phi} = \frac{3}{2}b^2(1 + \Lambda\alpha), \quad \frac{\partial\beta^\chi(\phi^*)}{\partial\chi} = \frac{3}{2}c^2(1 + \Lambda\alpha). \quad (5.11)$$

and

$$\frac{\partial\beta^\phi(\phi^*)}{\partial\chi} = \frac{\partial\beta^\chi(\phi^*)}{\partial\phi} = \frac{3}{2}bc(1 + \Lambda\alpha), \quad (5.12)$$

The equations (5.8) becomes

$$U \frac{d\phi}{dU} = \frac{\partial\beta^\phi(\phi^*)}{\partial\phi}(\phi - \phi^*) + \frac{\partial\beta^\phi(\phi^*)}{\partial\chi}(\chi - \chi^*) + \dots \quad (5.13)$$

$$U \frac{d\chi}{dU} = \frac{\partial\beta^\chi(\phi^*)}{\partial\phi}(\phi - \phi^*) + \frac{\partial\beta^\chi(\phi^*)}{\partial\chi}(\chi - \chi^*) + \dots \quad (5.14)$$

Substituting the explicit beta functions (5.11) and (5.12) into eqs. (5.13) and (5.14), summing up one another and integrating out the resulting equation one can find that the running couplings $\phi^i = (\phi, \chi)$ vary according to the formula

$$\phi^i = \phi^{i*} + \frac{c^i}{U}, \quad (5.15)$$

where we have used the fact that $b^2 + c^2 = -2/3(1 + \Lambda\alpha)$. Again, we have here a dual field theory exhibiting an weakly coupled regime at high energy. The running couplings ϕ, χ , are fixed, i.e. $\phi^i = \phi^{i*}$, as $U = e^A \rightarrow \infty$ in the slice $y > 0$ of the AdS_4 brane that we have found earlier.

Let us now return to the discussion about gravity localization for this brane solution. Our former calculation shows that there exist only *massive gravity* on the spectrum. The

gravity localization is favored as long as a very light graviton mode emerges, such that gravity is locally localized [10, 38–41], although asymptotically the warp factor diverges. This agrees with the well-known fact that there is no normalizable graviton zero mode as the warp factor diverges [1, 7, 8, 45, 46, 10, 40, 41].

6. Ending comments

In this work we have shown how to write a first-order formalism to braneworld scenarios which include the possibilities of the brane to have AdS, Minkowski, and dS geometry. The crucial ingredient was the introduction of two new functions, $W = W(\phi)$ and $Z = Z(\phi)$ from which we could express both A and ϕ in terms of first-order differential equations, for the potential engendering very specific form. The importance of the procedure is related not only to the improvement of the process of finding explicit solution, but also to the opening of another route, in which we can very fast and directly write the warp factor once $W(\phi)$ and $Z(\phi)$ are given. As we have shown, the present investigations seem to open several distinct possibilities of study.

The issue concerning the gauge/gravity duality is interesting, and the first-order formalism here developed for AdS, Minkowski, or dS geometry can easily be used to find the renormalization group flow of the holographically dual field theory. By considering some models given in terms of specific superpotentials, we have shown that there are interesting “bent” brane solutions with negative cosmological constant (AdS_4 branes) that can play the role of nice gravity duals. The RG flow shows that UV stable fixed point of a dual field theory can be treated perturbatively at high energy, and just like QCD it may develop ‘asymptotic freedom’. Such “bent” branes may give a dual gravitational description of RG flows in supersymmetric field theories living in the curved spacetime of the brane world-volume [23]. Furthermore, as we have shown in an explicit example, AdS_4 branes seem to exhibit improved infrared behavior. Other examples include *asymmetric branes* that are asymptotically $M_5 - AdS_5$ spaces. We found that in these theories there exists a ‘natural’ UV cut-off on the running coupling in the Minkowski (M_5) side. All the brane solutions we found here are connecting two different vacua. A natural continuation is to look for solution connecting critical points other than vacua, such as local maxima, saddle points, and so on [22] to look for other gravity duals.

Evidently, in addition to the subject of gravity localization on thick branes, the interest on the subject broadens with the application of the method to investigate new gravity duals and renormalization group flow of the holographically dual field theory, since now we can easily find the flow equations for brane models with arbitrary cosmological constant and bulk scalar fields that can be identified with running couplings in the dual field theory.

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